Can One "Hear" the Shape of a Sapphire Whispering Gallery Mode Resonator?

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Whispering gallery modes (WGM) low loss dielectric resonators have found widespread applications in the fields of microwaves, millimeter waves and optics. A wide variety of resonator shapes such as disks, rings, toroids or spheres, can sustain the propagation of WGMs. The rotational symmetry of these resonators leads to a degeneration of the solutions of the Helmholtz equation, yielding the two-fold degeneracy of the WGMs. Any deviation from the resonator's symmetry, induced by a defect, lifts this degeneracy, and a resonance line splitting is generally observed[1]. Such defects can manifest as geometrical imperfections, particles adhering to the resonator surface or inhomogeneities within the dielectric bulk. For many applications, this phenomenon proves detrimental. In the case of a microwave WGM resonator oscillator, frequency jumps between the two degenerate modes can compromise the oscillator stability. Conversely, in photonics WGM resonators, the degeneracy and line splitting phenomenon can be harnessed for sensing nanoparticles, such as contaminants or viruses. Regardless of the application, enhanced control over mode splitting and a deeper understanding of its origin would be beneficial.

Until now, the prevailing guess attributes the mode splitting observed in sapphire microwave WGM resonators to randomly situated defects affecting the resonator's geometry or homogeneity. Contrary to this widespread belief, our paper challenges this idea and demonstrates that mode splitting originates from a precisely determined geometrical defect. This defect is consistently observed in all sapphire resonators, irrespective of their origin. Notably, it aligns with the 6-fold symmetry of the sapphire crystal and correlates with the distinct crystallographic planes, which respond differentially during the sapphire machining process. As a secondary outcome of our study, we offer here a modest answer to the question *Can One Hear the Shape of a Drum?* posed by M. Kac in 1966 in his renowned article[2]. It took mathematicians approximately thirty years to provide a general negative answer to this question. While we do not delve into the intricacies of this complex mathematical problem in this article, we show how the phenomenon of mode-splitting observed in a microwave whispering gallery mode sapphire resonator allows for the deduction of its deviation from the ideal cylindrical form.

[1] P.-Y. Bourgeois and V. Giordano, "Simple model for the mode-splitting effect in whispering-gallery-mode resonators," *IEEE Trans. on MTT.*, vol. 53, no. 10, pp. 3185–3190, Oct. 2005.

[2] M. Kac, "Can one hear the shape of a drum?" *Amer. Math. Monthly*, vol. 73, no. 4P2, pp. 1–23, 1966.